HARMONIC SERIES.

HARMONIES OF THE CHEMICAL ELEMENTS.

By B. B. SMYTH.

The development of harmonic forms from arithmetical and figurate series; and the study of harmonical series in the phyllotactic arrangement of leaves, scales, etc., of plants and their fruits; in the waves, tones, and velocities of music, light, and electricity; in the distances, weights, and movements of the planets and their satellites; in organic chemistry; and the finding of harmony everywhere in nature has led to researches to determine what harmony there should be in the constitution of the primary elements and in their respective properties.

Earliest researches were happily made in wrong directions, resulting in negative knowledge, which serves as a lamp to positive knowledge. The atomic weights of all the elements were first resolved into their prime factors, with a view to determining a relation between their weights and valences. The result was generally negative.

One difficulty lies in the fact that a slight error in the determination of the atomic weight of an element would lead to a conclusion entirely erroneous.

For instance, the atomic weight of gold is often given as 197. It is also sometimes given as 199. Each of these is a prime number; and if either were correct, and the atomic weights were in exact multiple proportion, gold could have only one valency, namely monovalency. But gold is also trivalent. Yet the factor 3 does not enter into either of these numbers. If the atomic weight were shown to be 198, then the metal might be divalent or even hexavalent as well as trivalent.

In a few cases (as, for instance, Ca 40, Cd 112, and Hg 200, which are divalent; Al 27, Ga 69, and Sm 150, which are trivalent; C 12, and Ti 48, which are tetravalent; As 75, and Sb 120, which are pentavalent; Mo. 96, and U 240, which are hexavalent), the atomic weight appears to be in multiple proportion to the valency; but in most cases there is no such concordance. If this were true, N 14, Fe 56, and Sn 119, should be heptavalent; and Ca 40, Mn 55, Br 80, Te 125, Sm 150, Tb 160, Pt 195 and Hg 200 should be pentavalent. Similarly, O 16, Ca 40, Br 80, Sb 120, Tb 160 and Hg 200 should be tetravalent if not pentavalent.

This is a field that has been worked over thoroughly by many eminent chemists in the last forty years; yet at this late day there are golden grains to be garnered even by a mathematician; and, whether new or old, these ideas are presented to show the harmonies that exist in the constituent properties of the several chemical elements.

On these charts I have arranged the elements in octaves as done thirty years ago by Newlands, and much better done four years later by the Russian chemist Mendeléeff, that being the most rational method yet devised for a classification of the chemical elements. The arrangement of Dr. Charles Skeele Palmer, of the University of Colorado, is an admirable one, and a vast improvement over that of Mendeléeff. I have, however, modified that arrangement to some extent to accord with recent discoveries in inorganic chemistry.

According to their specific gravities, the elements, after the first two octaves, are arranged in hecdecades. In the first two octaves the specific gravities increase in a general way to the middle of each octave, then decrease to the end. The common difference in the atomic weights of these two octaves is 2.

After the first two octaves the specific gravity increases somewhat irregularly to the middle of each hecdecade, then decreases similarly to the end. The common difference in atomic weights is 3. Irregularities in atomic weight do not change this mean until the fourth hecdecade, where there seems to be a slight reduction.

Each hecdecade consists of two full octaves, an accrescent, in which the specific gravities of the several successive elements increase to the end of the octave; and a decrescent, in which the specific gravities regularly fall off from the beginning of the octave to the end.

The first table contains an additional—a hypothetical—octave, containing the one element hydrogen. Whether this octave shall ever be filled through future discoveries, or by placing therein some of the elements already known and wrongly placed in some other octave, remains to be seen.

The atomic weights are, unless otherwise specified, as published by Dr. F. W. Clarke, chief chemist U. S. geological survey, January 1, 1894.

HYDROGEN (HYPOTHETICAL) OCTAVE.

Valence.	Harmonic series.		Element ...	Atomic weight. (O = 16) ...	Difference ...	Specific gravity. (H_2O = 1) ..	Variation ...
	At. Wt.	Spec. Grav.					
I	1	.35	H	1	0	0.025	....
II	1.5	.54	....	....	....	....	....
III	2.25	.82	....	....	....	....	....
IV	3	1.22	....	....	....	....	....
III, V	4	.93	....	....	....	....	....
II, VI	5	.69	....	....	....	....	....
I, VII	6	.52	....	....	....	....	....

LITHIUM OCTAVE.

Valence.	Harmonic series.		Element....	Atomic weight. (O = 16)....	Difference....	Specific gravity. (H_2O = 1)..	Variation....
	At. Wt.	Spec. Grav.					
I	7	.58	Li	7	0	0.58	.00
II	9	.90	Gl ?	9	0	1.85	+1.05
III	11	1.36	B ?	11	0	2.61	+0.97
IV	13	1.95	C	12	—1	2.25*	+0.15
III, V	15	1.48	N	14	—1	0.88	—0.40
II, VI	17	1.11	O	16	—1	1.11	.00
I, VII	19	.83	F	19	0	—	....
VIII	21	.65	....	....	....	....	....

* Graphite.

Between Boron and Carbon is a half-step or minor step; between Oxygen and Fluorine is a step and a half or major step. There is room for an additional element between O and F; possibly room for elements with C and N.

Boron and Glucinum seem to differ widely in some respects from the require-

ments of the positions they occupy; but in the absence of positive knowledge, they are allowed to remain in the series as placed by Mendelóeff.

SODIUM OCTAVE.

Valence.	Harmonic series. At. Wt.	Harmonic series. Spec. Grav.	Element...	Atomic weight. (O = 16)...	Difference...	Specific gravity. (H_2O = 1)..	Variation...
I	23	.97	Na	23.1	+0.1	0.97	0
II	25	1.50	Mg	24.3	−0.7	1.75	+.17
III	27	2.22	Al	27	0.	2.68	+.21
IV	29	2.84	Si	28.4	−0.6	2.48	−.12
III, V	31	2.35	P	31	0.	2.34	0
II, VI	33	2.01	S	32.1	−0.9	2.07	+.03
		....	....	....	....	....	....
I, VII	35	1.33	Cl	35.5	+0.5	1.33	0
VIII	37	1.00	....	....	....	....	....

The differences in atomic weight between Na and Mg, between Al and Si, and between P and S, are minor steps; the differences between Mg and Al, between Si and P, and between S and Cl, are major steps. They recur regularly.

POTASSIUM HECDECADE.

Valence.	Harmonic series. At. Wt.	Harmonic series. Spec. Grav.	Element...	Atomic weight. (O = 16)...	Difference...	Specific gravity. (H_2O = 1)..	Variation...
I	39	1.05	K	39.1	+0.1	.87	−.17
II	42	1.79	Ca	40	−2	1.70	−.05
			....	....	....	....	....
III	45	2.69	Sc	44	−0.1	....	....
IV	48	3.69	Ti	48	0	....	....
III, V	51	5.00	V	51.4	+0.4	5.87	+.17
II, VI	54	6.60	Cr	52.1	−1.9	6.81	+.03
II, IV	57	7.20	Mn	55	−2	7.20	
(VII)		8.00	Fe	56	−1	8.00	
II, IV	60	8.90	Ni	58.9	−1.1	8.90	
VIII		9.09	Co	59.7	−0.3	8.96	−.
II, I	63	8.50	Cu	63.6	+0.6	8.90	+.
II	66	7.40	Zn	65.3	−0.7	7.12	−.
III	69	6.43	Ga	69.1	+0.1	5.95	−.
IV	72	5.60	Ge	72.3	+0.3	5.47	−.
III, V	75	4.87	As	75	0	4.71	−.
II, VI	78	4.23	Se	79	+1	4.31	+.
I, VII	81	3.37	Br	80	−1	3.19	−.

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Ni and Co appear to be doublets or twins. So, also, appear Mn and Fe. They are complementary to each other. Ca and Se are each one side of its systematic position. Their complementary elements are yet to be discovered.

RUBIDIUM HECDECADE.

Valence.	Harmonic series. At. Wt.	Harmonic series. Spec. Grav.	Element.	Atomic weight (O = 16).	Difference.	Specific gravity (H_2O=1)	Variation.
I	84	1.40	Rb	85.5	+1.5	1.52	+.08
II	87	2.38	Sr	87.6	+0.6	2.50	+.05
III	90	3.58	Yt	89.1	—0.9	....	....
IV	93	4.90	Zr	90.6	—2.4	4.15	—.15
III, V	96	6.67	Cb	94	—2	6.75	+.01
II, VI	99	8.80	Mo	96.1	—3	8.60	—.02
II, VII	102	10.82	Ru	101.6	—0.4	11.44	+.06
II, IV / VIII	105	12.12	Rh	103	—2	12.10	.00
			Pd	106.6	+1.6	11.85	—.02
I	108	10.90	Ag	107.9	—0.1	10.50	—.04
II	111	9.50	Cd	112	+1	8.65	—.09
III	114	8.23	In	113.7	—0.3	7.42	—.09
IV	117	7.14	Sn	117.8	+0.8	7.25	+.01
III, V	120	6.21	Sb	120	0	6.70	+.08
II, VI	123	5.40	Te	125	+2	6.20	+.15
I, VII	126	4.30	I	126.9	+0.9	4.90	+.04

CESIUM HECDECADE.

Valence.	Harmonic series. At. Wt.	Harmonic series. Spec. Grav.	Element.	Atomic weight (O = 16).	Difference.	Specific gravity (H_2O = 1).	Variation.
I	129	1.87	Cs	132.9	+3.9	1.88	00
II	132	3.18	Ba	137.4	+5.4	3.75	+.18
III	135	4.75	La	138.2	+3.2	6.10	+.29
III, IV	138	6.50	Ce	140.2	+2.2	6.70	+.03
			Nd	140.5	+2.5	....	....
III, (V)	141	8.89	Pr	143.5	+2.5	....	....
VI	144	11.64	....	....	....	....	....
VII	147	14.48	....	....	....	....	....
III, (IV) / (VIII)	150	16.16	(Sm	150	0)	....	....
III, (I)	153	13.97	(Da	154	+1)	....	....
III, (II)	156	12.20	(Gd	156.1	+0.1)	....	....
III	159	10.52	(Tb	160	+1)	....	....
IV	162	9.13	....	....	....	....	....
III, (V)	165	7.92	(Er	166.3	+1.3)	....	....
III, (VI)	168	6.89	(Tu	170.7	+2.7)	....	....
III, (VII)	171	5.50	(Yb	173	+2)	....	....

Between Cs and Ba is a major step; between Ba and La is a minor step; Ce and Nd appear to be twins; otherwise Nd is superfluous.

The entire hecdecade needs revision. The first four elements, whose specific gravities have been carefully determined, appear to have their atomic weights too high to correspond with their specific gravities. Especially is this true of Barium. If the harmonic series of atomic weights were increased three units, La would be the only element outside the limit of variation. Barium would then correspond exactly as to its atomic weight and specific gravity, owing to the recent redetermination of the atomic weight by Richards. Care was taken in this redetermination to eliminate all lighter metals of the same group, as Ca, Sr, etc. Effort should now be made to eliminate from Ba, La, and Ce, the heavier metals, II, III and IV of the Tantalum octave. This should bring them within the harmonic law.

The rest of the hecdecade is arranged simply according to atomic weight. The valency cuts no figure in the case. They are all represented as trivalent. It is inconceivable that all the elements of a hecdecade except the first two should be trivalent. None of them correspond with the specific gravities opposite to which they are placed. The specific gravity of Davydium (9.39) would show that it belongs, not where it is placed but further down the scale. The other properties, as valence, brittleness, melting point, electrical status, spectral lines, transparency to certain forms of radiant energy, etc., must determine its position. The same is true of the other elements.

(GOLD) HECDECADE.

Valence.	Harmonic series.		Element....	Atomic weight (O=16)....	Difference....	Specific gravity (H_2O=1)	Variation....
	At. Wt.	Spec. Grav.					
I	174	2.49					
II	177	4.26					
III	180	6.40					
IV	183	8.65					
III, V	186	11.86	Ta	182.6	—3.4	10.70	—.10
II, VI	189	15.80	W	184.9 187.1	—4.1	18.25	+.15
II, VII	192	19.30	Os	190.8	—1.2	22.48	+.17
II, IV (VIII)	195	21.55	Ir Pt	193.1 195	—1.9 0	21.80 21.50	+.01 00
III, I	198	17.86	Au	197.3	—0.7	19.30	+.08
I, II	201	15.60	Hg	200	—1	14.40	—.08
III	204	13.50	Tl	204.2	+0.2	11.90	—.12
IV	207	11.70	Pb	207	0	11.30	—.03
III, V	210	10.18	Bi	208.9	—1.1	9.82	—.03
II, VI	213	8.80					
I, VII	216	7.03					

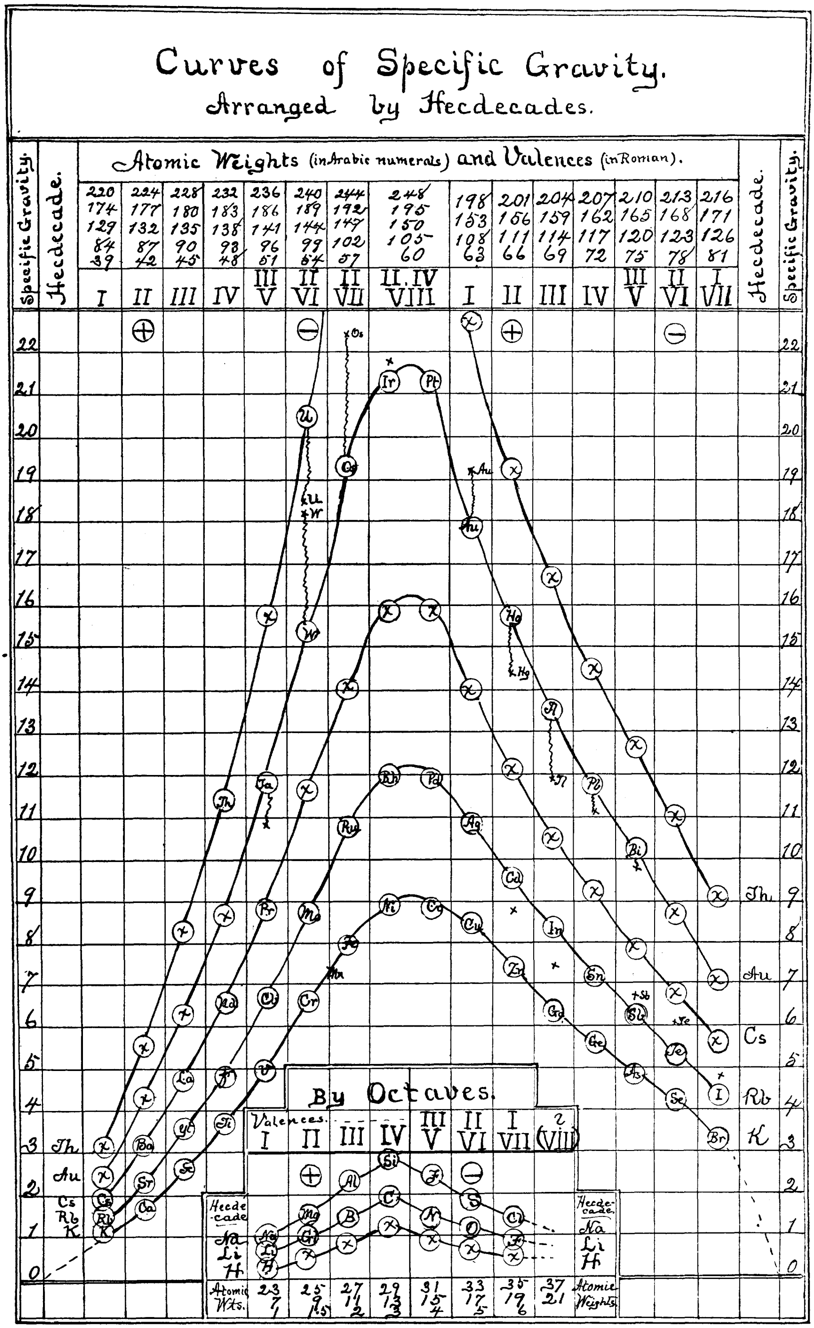
In all this hecdecade the atomic weights as determined seem to be too low to correspond with the specific gravities. This is especially true of Tungsten and Osmium.

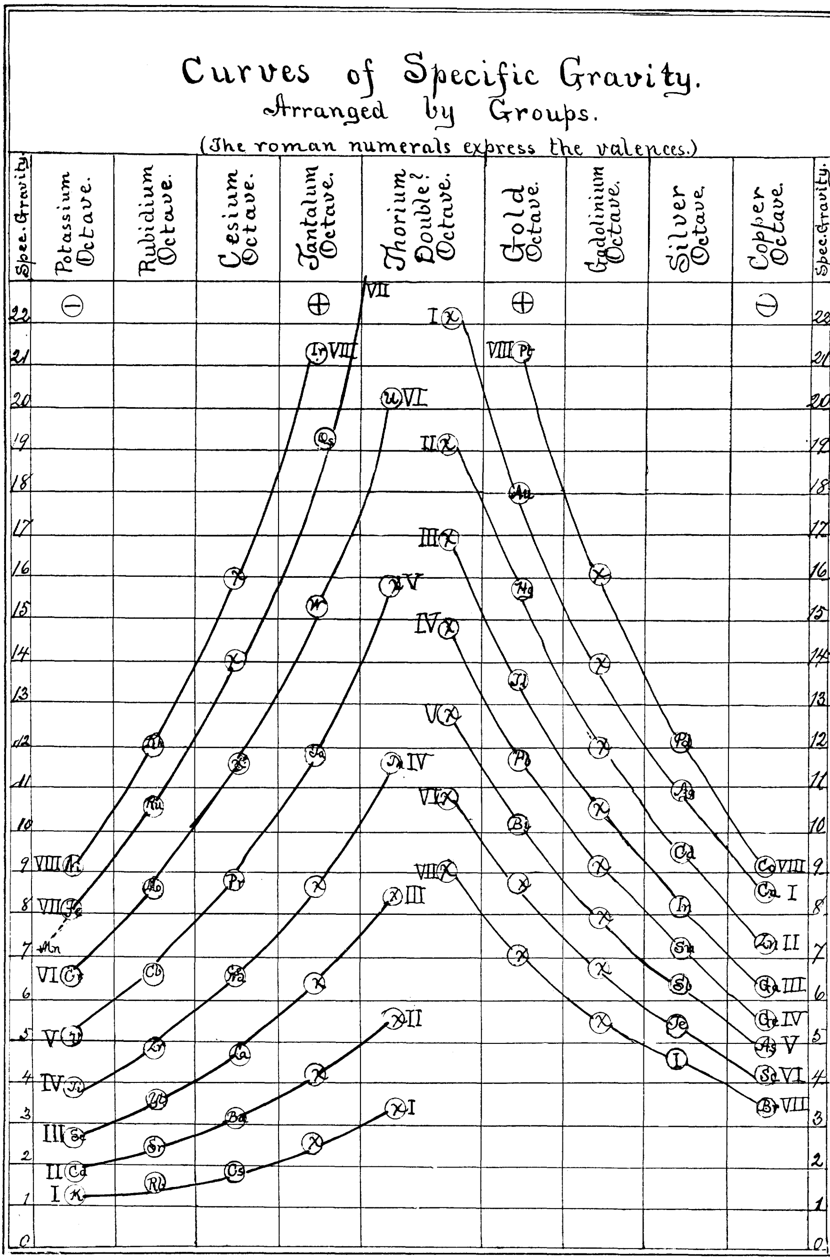
(THORIUM) HECDECADE?

Valence.	Harmonic series.		Element.....	Atomic weight (O=16)....	Difference....	Specific gravity (H_2O=1)	Variation....
	At. Wt.	Spec. Grav.					
I	220	3.11					
II	224	5.56					
III	228	8.33					
IV	232	11.23	Th	232.6	+0.6	11.23	.00
III, V	236	15.40					
II, VI	240	20.50	U	239.6	—0.4	18.40	—.10
I, VII	244	25.05					
VIII	248						

Not enough is known of this hecdecade to know whether the common difference in atomic weights is 3, 3½, or 4; nor whether it is a hecdecade of two octaves or something else. Whether the limit of chemical elements is reached in this hecdecade, or whether they continue indefinitely to increase in atomic weight and specific gravity as more new elements are discovered; or whether they will all at last be resolved into still simpler forms, remains for the future to tell.

I here present two graphic charts showing the curves of specific gravity, arranged first by hecdecades, second by groups.


Curves of Specific Gravity.
Arranged by Hecdecades.
Atomic Weights (in Arabic numerals) and Valences (in Roman).
Specific Gravity.
Hecdecade.
By Octaves.
Valences.
Atomic Wts.
Atomic Weights.



The rate of increase in specific gravity in the elements nearest the culminating point in the curve of each hecdecade is 34½ per cent., thus: Co (K VIII) 9.00; Rh (Rb VIII) 12.10; —— (Cs VIII) 16.24; and Ir (Au VIII) 21.80. Os (Au VII) is 22.40, which is still higher; but the specific gravity of

Os is excessive and not a fair average. Os may contain an undiscovered element (Th VII) of still higher atomic weight and specific gravity. The curves, however, are calculated on an increase of 33⅓ per cent. for the culminating point, thus: K VIII, 9.09; Rb VIII, 12.12; Cs VIII, 16.16; and Au VIII, 21.55. This places Co 1 per cent. below and Ir 1 per cent. above the culmination. According to this calculation, if there were an element at Th VIII its specific gravity would be 28.73; but, according to those that already exist, as Co, Rh, and Ir, in which the rate of increase is 34½ per cent., the specific gravity would be 29.25. In the case of the twin metals, Ni, Pd, and Pt, standing with Co, Rh, and Ir, at the culmination of the curves, the variation is so slight it need not be taken into account.

The average rate of increase in specific gravity of the several successive members of each of the accrescent octaves, while not uniform, is approximately 36 per cent. The decrease in specific gravity of the several successive members of the decrescent octaves averages 13⅓ per cent., or, computing backward and upward from the lowest element in each octave, the increase is 15 per cent.

There are eight groups in the accrescent octaves and seven in the decrescent, fifteen in all. The rate of increase in specific gravity in the several elements of each group in the accrescent octaves is strictly 33⅓ per cent. The members of the Thorium octave are an exception; the increase is only 15 per cent. The increase of specific gravity in the several elements of each group in the decrescent octaves is 28 per cent. The Thorium hecdecade is no exception; there are no known elements in the decrescent octave of the Thorium hecdecadal series. Hence there is no decrescent octave; and the term hecdecade is simply used in this case for convenience and comparison.

Harmonies of the Hecdecades.

The atomic weights, while not strictly a uniform arithmetical series, are nevertheless harmonic to a degree. Like the tones in music, there are whole steps and half steps, major steps and minor steps, as already shown, and occasionally doublets or twins. Perhaps when all the elements in nature have been discovered, there will be a "chromatic" series in each octave. That these atomic weights are harmonic is readily shown by placing them in the form of a square, thus:

Potassium Hecdecade.

				238.9
K I 39.1	Ga III 69.1	As V 75	Mn VII 55	238.2
Se VI 79	Cr VI 52.1	Ti IV 48	Ni IV 59.3 Co	238.4
Fe IV 56	Ge IV 72.3	Zn II 65.3	Ca II 40	233.6
Cu I 63.6	Sc III 44	V V 51.4	Br VII 80	239.0
237.7	237.5	239.7	234.3	236.5

Besides the sums in the margin, obtained by addition of the lines, columns, and diagonals, we obtain the following sums from this square: The four corners, K, Mn, Cu and Br equal 237.7; the central square, Cr, Ti, Ge and Zn equal 237.7; parallelograms, Se, Fe, Co and Ca equal 234.3; Ga, As, Sc and V equal 239.4; diagonal quadrats, Ga, Fe, V and Co equal 235.8; As, Se, Sc and Ca equal 238; rhombs, K, Ge, Ti and Br equal 239.4; Mn, Zn, Cr and Cu equal 236; rhomboids, K, Sc, As and Br equal 238.1, etc., etc. In all of these sums the greatest is 239.7, and the least is 233.6, a difference of 6.1, equal to 2½ per cent., or 1¼ per cent. on each side of a fixed mean.

By rejecting fractions and bringing Ca within the limit of variation and changing slightly the elements in the third column, we obtain the following:

POTASSIUM HECDECADE. 237

K I 39	Ga III 69	As V 74	Mn VII 55	237
Se VI 79	Cr VI 52	Ti IV 47	Ni IV 59 Co	237
Fe IV 56	Ge IV 72	Zn II 66	Ca II 43	237
Cu I 63	Sc III 44	V V 50	Br VII 80	237
237	237	237	237	237

Here it may be seen that, though the atomic weights are not an arithmetical series, all sums, taken as before, are equal, showing that the series is harmonic.

A similar result is obtainable from a similar placing of the atomic weights of the Rubidium hecdecade, thus:

RUBIDIUM HECDECADE. 417.9

Rb I 85.5	In III 113.7	Sb V 120	Ru VII 101.6	420.8
Te VI 125	Mo VI 96.1	Zr IV 90.6	Pd IV 106.6	418.3
Rh IV 103	Sn IV 117.8	Cd II 112	Sr II 87.6	420.4
Ag I 107.9	Yt III 89.1	Cb V 94	I VII 126.9	417.9
421.4	416.7	416.6	422.7	420.5

The greatest sum in this square is 422.7; the least is 416.6. The difference is 6.1, which is 1.4 per cent., or less than three-fourths of one per cent. on either side of a fixed mean.

CESIUM HECDECADE.

				611.9
Cs I 132.9	III 163	Tu V 170.7	Sm VII 150	616.1
Yb VI 173	VI 147	Ce IV 140.5 Nd	Da IV 154	614.5
IV 153	Er IV 166.3	Tb II 160	Ba II 137.4	616.7
Gd I 156.7	La III 138.2	Pr V 143.5	VII 175	612.8
615.0	614.5	614.7	616.4	614.9

The Cesium hecdecade similarly arranged gives results in which the greatest differences are less than one-half of one per cent. on either side of a fixed mean.

CESIUM HECDECADE.

				601.2
Cs 132	Tb 160	Er 166.3	Pr 143.5	601.8
Tu 170.7	Ce 140.2	Nd 140.5	Sm 150	601.4
146	164.2	Gd 156.1	135	601.3
Da 153	Ba 137.4	La 138.2	Yb 173	601.6
601.7	601.8	601.1	601.5	601.3

A rearrangement of the elements of this hecdecade, so as to separate Nd from Ce brings the results so nearly equal that the square is almost perfectly harmonic. Addition of the corner and central squares, the parallelograms, diagonal quadrats, etc., scarcely increases the variation from a fixed mean.

In the gold hecdecade there are only ten elements. The square must be filled up with the atomic weights of six unknown elements, thus:

GOLD HECDECADE.

				777.7
I 174	Tl III 204.2	Bi V 208.9	Os VII 190.8	777.9
VI 213	VI 187.1	Ta V 182.6	Pt IV 195	777.7
Ir IV 193.1	Pb IV 207	Hg II 200	II 177	777.1
Au I 197.3	III 179.5	W VI 184.9	VII 216	777.7
777.4	777.8	776.4	778.8	777.1

Thus arranged, the gold hecdecade forms a square more perfectly harmonic than any of the lower hecdecades.

It should be easy, from a study of these hecdecadal tables and graphic charts to predetermine the characteristics and properties of any element before its discovery with a greater degree of accuracy than could have been done by Mendeléeff's octaves. I confidently await such discovery.